1. Each of two identical spherical volumes with radius R is uniformly charged with a positive charge Q throughout the volume. One sphere is centered at the origin and the other at $x=2 R$. Find the magnitude and direction of the net electric field due to the two charged spheres at the following points:
(a) (10 points) $x=0$ and $y=0$;
(b) (10 points) $x=R / 2$ and $y=0$;
(c) (10 points) $x=3 \mathrm{R}$ and $\mathrm{y}=0$;
(d) (10 points) $x=0$ and $y=2 R$.

2. A conducting bar of length $L=0.8 \mathrm{~m}$, mass $\mathrm{m}=0.9 \mathrm{~kg}$, is free to slide without friction on a horizontal rails. There is a uniform magnetic field $\mathrm{B}=1.5 \mathrm{~T}$ directed into the plane. A 12 V battery is connected to the rails through a switch S . The bar has resistance $\mathrm{R}=5 \Omega$. The resistance of other parts of the circuit can be neglected. The switch is closed at $\mathrm{t}=0$.
(a) (15 points) Immediately after the switch is closed, what is the magnitude and direction of acceleration of the bar?
(b) (10 points) What is the acceleration of the bar when its speed is $2 \mathrm{~m} / \mathrm{s}$ ?
(c) (10 points) The velocity of the bar reaches a terminal value (terminal velocity). Explain why the bar should reach a terminal velocity, and find the value of the terminal velocity.
3. A circular wire of radius R lies on a horizontal table and carries a current $I$. Point A is at the center of the circle and point C is on the circle.
(a) (10 points) Find the direction and magnitude of the magnetic field at point A;
(b) (10 points) The wire is now unwrapped into a straight line segment with point C in the middle. The straight line segment, still carrying the current I , is perpendicular to the line segment AC. Find the magnitude of the magnetic field at point A again.
(c) ( 5 points) Which field is greater? And explain why so.

4. (20 points) A metal bar with mass $\mathrm{m}=0.5 \mathrm{~kg}$, length $L=1.5 \mathrm{~m}$ and resistance $\mathrm{R}=$ $10 \Omega$, rests horizontally on conducting rails that connect the bar to a circuit through a switch $S$. The source of $e m f$ has an adjustable $\varepsilon$. The metal bar itself is in a uniform, horizontal magnetic field of 2T as indicated in the figure below. Find the value of $\varepsilon$ such that after $S$ is closed, the magnetic force on the metal bar just cancels the gravitational force on the bar (acceleration due to gravity: $g=9.8 \mathrm{~N} / \mathrm{kg}$ $=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ).

5. (20 points) In the following circuit, find the current through each of the three resistors. You may ignore the internal resistance of the emf sources.

6. For the capacitor network shown in the following figure, the potential difference across $a b$ is 12 V .
(a) (15 points) Find the total energy stored in this network;
(b) ( 10 points) Find the energy stored in the $3 \mu \mathrm{~F}$ capacitor.

7. An infinitely large flat sheet is uniformly charged with $\sigma_{0}=+4 \times 10^{-7} \mathrm{C} / \mathrm{m}^{2}$. The thickness of the sheet can be neglected. A circular hole with a radius $\mathrm{r}=1 \mathrm{~cm}$ is created in the sheet by removing a disc-like portion of the same radius from the original sheet as shown below. Let the sheet be in the $x-y$ plane, and the origin of the coordinate frame at the center of the hole.
(a) (10 points) Find the magnitude and direction of the electric field $\overrightarrow{\mathrm{E}}(\mathrm{z})$ at $\mathrm{z}>0$ on the z-axis in terms of $\sigma_{0}, r, \varepsilon_{0}$, and unit vectors $\hat{i}, \hat{j}$, and $\hat{k}$.
(Hint: superposition principle)
(b) (10 points) Show that the electric potential as a function of z on z -axis referenced to the potential at the origin is (see below for a useful integral)

$$
\begin{equation*}
\mathrm{V}(\mathrm{z})-\mathrm{V}(0)=-\frac{\sigma_{0}}{2 \varepsilon_{0}}\left(\sqrt{\mathrm{z}^{2}+\mathrm{r}^{2}}-\mathrm{r}\right) \tag{1}
\end{equation*}
$$

(c) (15 points) An electron with charge $\mathrm{q}_{\mathrm{e}}=-1.6 \times 10^{-19} \mathrm{C}$ and mass $\mathrm{m}_{\mathrm{e}}=9.1 \times 10^{-31} \mathrm{~kg}$ is now at the origin with an initial velocity $\overrightarrow{\mathrm{v}}_{0}=2 \times 10^{7} \mathrm{~m} / \mathrm{s} \hat{\mathrm{k}}$. How far on the zaxis will the electron travel before it comes to a stop and begins to turn back? (hint: you can use Eq. (1) from Part b, and the constant $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{m}^{2}$.)


Useful integral: $\int_{0}^{\mathrm{b}} \frac{\mathrm{xdx}}{\sqrt{\mathrm{x}^{2}+\mathrm{a}^{2}}}=\sqrt{\mathrm{b}^{2}+\mathrm{a}^{2}}-\mathrm{a}$

